A MATHEMATICAL APPROACH TO MANUFACTURING PROCESS AND SYSTEMS MODELING USING GEOMETRIC PROGRAMMING

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ABSTRACT

Recent advances in composite materials and manufacturing processes for commercial aircraft have brought disruptive change to the industry. It has been challenging to quantify the business impacts of coupled changes in vehicle performance and production processes of new composites systems. We present an approach to integrate nonlinear models for manufacturing processes, production systems, and design using Geometric Programming.

The models capture the interconnections between manufacturing processes, production systems, and supply chain, and financial performance in a single mathematical framework. Trade-offs across disparate disciplines are easily quantified. Sensitivities are obtained without requiring simulations. Parameter uncertainties are propagated to understand risks to performance, making this approach particularly suited to composite manufacturing.

In a case study, we show how major contributors to manufacturing cost are estimated with minimal design data. We present an example trade study for different composite technologies in manufacturing fuselage sections, showing how different manufacturing processes affect production system design and part costs. Sensitivities are used to show the major cost-drivers from design inputs.

1. BACKGROUND

Composite materials and structures play an increasingly important role in industries as diverse as aerospace to construction. Composites offer a vastly improved strength-to-weight ratio over more traditional materials. However, production with composites comes with its own set of challenges. Great progress has been achieved in understanding new materials, understanding how processing influences its properties, and developing equipment to process it more efficiently and reliably. A key issue that remains is accurately predicting associated production costs. This is particularly challenging since composite production is a highly serial process, where quality issues cannot easily be compensated through man-hours alone, and because nonlinear process

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behavior and quality requirements lead to variation, which is difficult to capture with existing production planning tools.

New processes and new designs require better tools for predicting and planning production systems. At the same time, little is known about the required production system at the beginning of the design process so it is difficult to clearly find the impact of improved processes or designs on manufacturing costs. Furthermore, the magnitude of the impact of a process or design change can be dependent on the production system itself.

This report offers an early introduction to a new method under development for enhanced composite manufacturing cost modeling for early design. Such a cost model aids researchers to express the business value of developing and integrating advanced processes. Or, in the inverse case, the models can be used to focus research and development on high-impact areas of current and future products and production systems. This report highlights, through a case study in aerospace, the advantage of uniting predictive technical models and descriptive financial models under a single mathematical framework.

1.1 Overview of Prior Art

Current approaches to manufacturing cost modeling and cost estimation generally fall into "top-down" and "bottom-up" categories [1][2]. Figure 1 shows the differentiating factors between the two types of estimating methods.

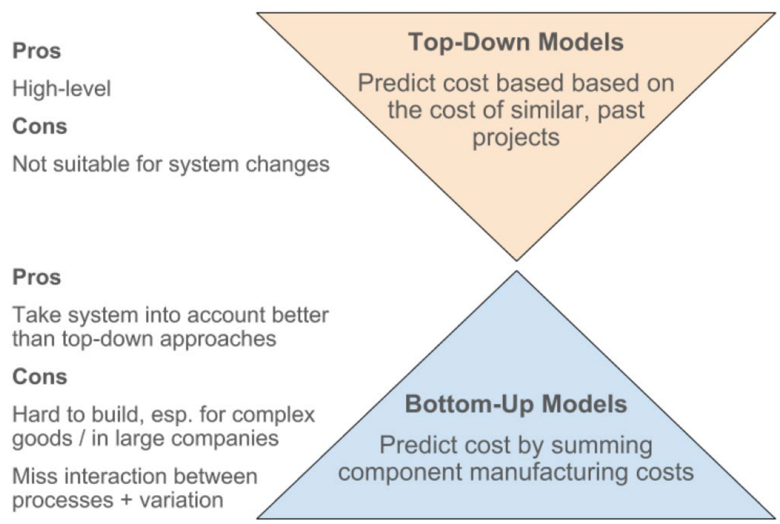

Figure 1. Overview of general approaches to cost modeling.

"Top down" methods use data from previous products to estimate the production cost of a new product based on the similarities or differences to previous productions. However, if a new product differs too greatly from previous products or production processes, top-down methods can fail to adequately estimate costs.

On the other hand, "bottom-up" methods consider the cost of each individual part of a product and the time to assemble the finished product from all of the constituent parts and subassemblies.

These methods can be very accurate when substantial design data are available, but bottom-up methods are difficult to implement effectively during early phases of design while there are still many unknown factors in the design.

Finally, neither top-down nor bottom-up cost estimating methods explicitly consider the behavior of the production system itself. Discrete Event Simulation is the tool most commonly employed for predicting the performance of a production system.

2. METHOD

The core of the issue is having a technical cost modelling tool which can be applied to early phases of design, where cost is most easily influenced, but which captures advancements in manufacturing and fabrication processes. Top-down techniques, based on historical data, may not appropriately capture cost changes arising from alterations to manufacturing processes and bottom-up techniques may require more detailed data than are available during early stages of design.

In many cases, significant manufacturing engineering efforts significantly lag behind the design of the product. Lacking important production system data, manufacturing costs can be particularly onerous to calculate. This is especially the case when tooling and production equipment are capital-intensive because the required number of machines and tools may not be known.

This cost modeling method uses Geometric Programming (GP) optimization to form optimal preliminary designs of the production system based on input design and process data and use these preliminary production system designs for estimating overall costs. This method can consider non-recurring capital costs of the production system, unit labor content, and recurring production costs.

2.1 Geometric Programming (GP)

Geometric Programming (GP) is a mathematical framework for constrained optimization problems. A notable feature of GP when compared to other optimization techniques is the ability to express nonlinear constraints but still be transformed into a convex optimization problem. Solving a convex constrained optimization problem has a number of nice mathematical features which include guaranteed global optima, robust solution techniques that don’t require initial guesses, and fast solution speeds [3]. While there are a number of phenomena which are highly non-convex, the techno-economic features relevant to design is dominated by the convex behavior of “diminishing marginal returns.”

The trade-off is that the GP formulation imparts certain restrictions on the mathematical form of the models which can be implemented. The models which are optimized must be expressed as posynomial inequalities. Equation 1 shows a posynomial inequality form of a constraint in a GP where x_n are the decision variables (variables to be optimized) and c_k and a_{nk} are constants (input parameters).

$$1 \geq \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \ldots x_n^{a_{nk}}, \qquad c_k > 0 \forall k, \quad a_{nk} \in \mathbb{R} \forall n, k \tag{1}$$

Though this form may appear to be restrictive, research shows broad applicability to engineering problems ranging from electronics [4], through water utility control [5], and aircraft design [6]. In all of these cases the key analytical design models were exactly formulated as posynomial constraints or represented through very accurate posynomial fits to experimental data.

Since the method is analytical, cost sensitivities are obtained for every input for a single solve of the underlying optimization problem. The sensitivities estimate the change in the objective if an input is changed *and* the model is optimized. This capability allows sensitivity analyses to scale gracefully as the problems grow in variables as the method is not beset with the same combinatorial curse that hampers the scaling of perturbation-based sensitivity analysis methods.

2.2 Evaluating Cross-Functional Trade-Offs

Modeling production costs and designing production systems rely on balancing trade-offs between very disparate functions within manufacturing enterprise: design, finance, production engineering, and supply chain functions. Figure 2 shows some of the interactions between these functions which must be evaluated in order to estimate production costs. In the past, these trade-offs have been rarely evaluated simultaneously. This cost modeling method allows not only for simultaneous evaluation of all the trade-offs, but also enables simultaneous optimization of the key trade-offs while optimizing the business case around cost.

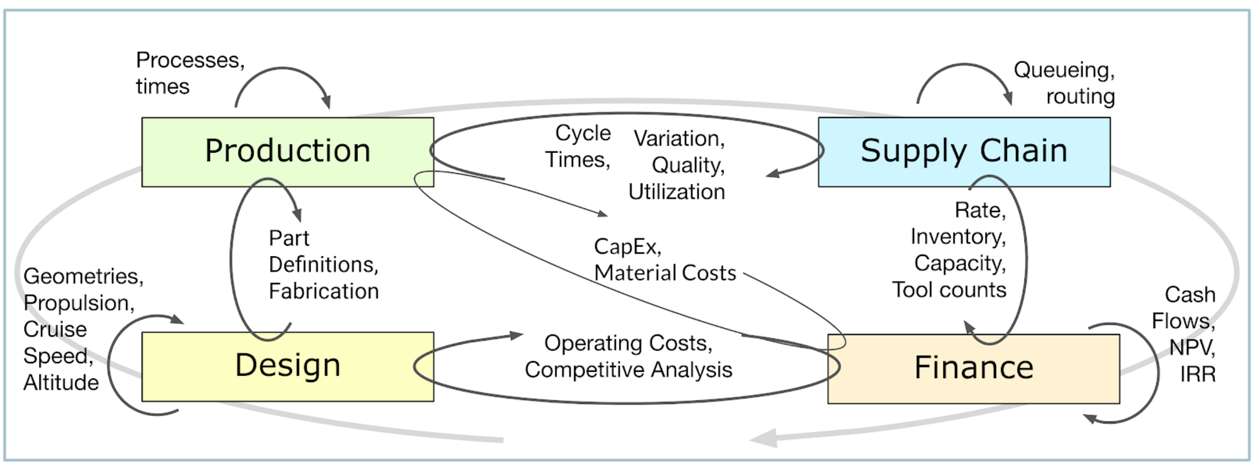

Figure 2. Production cost involves complex trade-offs among several interconnected functions across a manufacturing enterprise.

The core of the production system modeling is understanding how long it will take to execute certain processes for an input design and how material flows through the system. This model implements predictive technical models for manufacturing performance, based on design inputs, predictive technical models for the production system, and descriptive financial models.

All of the “models” which comprise the method are systems of mathematical equations which approximate the behavior of production processes, factory performance, and financial systems. These equations are reformulated as constraints in a GP and optimizing the GP trades-off all of the decision variables and solves all of the predictive models implemented as constraints.

2.3 Process Models

Process models are used in this method to predict process cycle times as a function of design inputs and process inputs. The method can use a combination of publicly available mathematical process models in literature, mathematical models based on sources in literature but modified with parameters which are proprietary to a particular organization, and finally models where both the equations and parameters are proprietary to an organization.

2.4 Financial Models

Financial models can be very specific or proprietary to a particular organization but there are general principles which are widely applicable to finding costs. The method can consider costs for a total program or costs on a unit bases. As well, the method accounts for non-recurring costs, such as equipment capital costs, and recurring costs, such as material usage and labor content.

2.5 Production Models

This method allows for analytical models to be used in place of simulation tools for predicting performance of production lines. Little’s Law, Equation 2, is the most fundamental model in production systems (the “Newton’s Law”) and describes the relationship between inventory, rate, and flow time [7]. Flow time is defined as the total time required for a single part to make its way through the system. In a simple example of a checkout line at a grocery store, the “production rate” is how many customer per hour a cashier can process, the “inventory” is how many customers are waiting in line, and the “flow time” is how long a customer should expect to spend in the checkout from the time they arrived in line to the time they are departing after paying.

$$
\begin{aligned}
L = \lambda W \\
where: \\
L = \text{Inventory in System} \\
\lambda = \text{Production Rate} \\
W = \text{Total Flow Time}
\end{aligned}
\tag{2}
$$

Production systems can have vastly different configurations but Little’s Law always holds. However, the interaction among inventory, flow time and production rate is complex and is highly dependent on the configuration of the production system. In production, the minimum flow time through the system is the time to complete all of the required processes which can predicted from the predictive process models with design inputs. Beyond this minimum, additional flow time arises from waiting for resources, rework, and other factors which also depend on the configuration of the production system. The task of balancing capacity and inventory in the production system is further complicated by variations present in the production system.

3. CASE STUDY

As a demonstration, the method is used to compare alternative composite manufacturing process for fuselage manufacturing: prepreg and resin infusion. The prepreg process is what is currently widely employed in fuselage fabrication. However, there are some who believe the infusion process would lower costs. This trade-off is explored using the presented models.

3.1 Scope

The motivation is to compare the costs of building production lines that utilize two different composite fabrication techniques: preimpregnated (prepreg) or resin infusion. The two techniques, differ by how the resin, which is ultimately cured to bond the carbon fiber together, is introduced to the production process. In the prepreg technique, carbon fibers that have already been preimpregnated with epoxy resin arrive as the raw material and are laid up in their slightly-tacky state. The prepreg stack is cured under heat and pressure in an autoclave oven. In the infusion method, carbon fibers are laid up dry and then resin is infused into the dry fiber stack under pressure until the fibers are saturated. The stack is then cured in an oven rather than an autoclave. Not needing an autoclave is among the chief reasons some expect the resin infusion process to have lower costs.

The cost model for this case study calculates the total non-recurring capital costs required for a factory to achieve the desired production rate. Given the design and production processes, the challenge is to add capacity and inventory to the factory in an optimal way such that required production rate is met at a minimum of cost.

The two different techniques are compared for constructing the same conceptual fuselage design. The comparative analyses are performed by implementing two cost models, one for each of the production alternatives, and comparing the solutions. The same fuselage geometry inputs and factory inputs are shared between the two, parallel models. Figure 3 shows the different models used to compare the processing techniques.

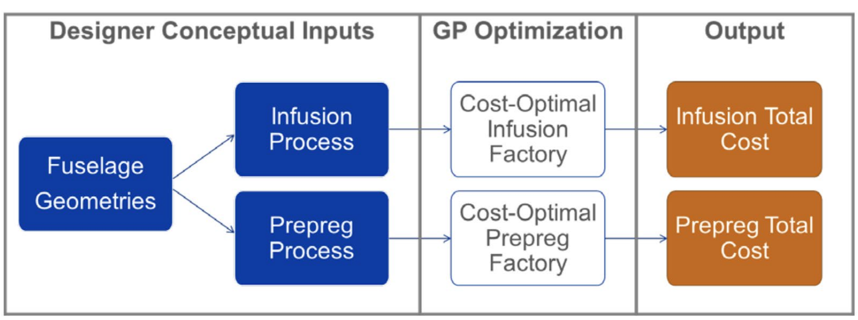

Figure 3. Implementation of comparative GP cost models considering the identical fuselage designs but manufactured in production lines executing different production processes.

3.2 Processes

The scope of the production cost model encompasses the processes where the two production alternatives most differ. The process flows for the prepreg and infusion methods are shown in Figure 4. Both methods require mold tooling which the fuselage section is fabricated upon. An empty tool enters the system where the different composite components are applied and cured. At the end of the system, the fuselage section is separated from the tool for further processing and the tool is returned to the beginning of the system to undergo the processes once again. The scope of the models is aligned with this tool flow to examine the number of tools as a driver for overall cost. Fuselage production continues identically after demolding.

A geometric program is optimized for each production approach: prepreg and infusion. Each of the processes shown in Figure 4 are modeled as their own cell in the production system. In this manner, the outlines of the techniques also serve as diagrams for the two conceptual factories that are modeled. The two formulations use the same models for total cost and for production line performance. The two production techniques are differentiated at the production cell level where there are different unit processes. Most of the unit process models were derived from the COSTADE models [8].

Figure 4. Outline of the prepreg and infusion techniques showing the constituent processes.

3.2.1 Prepreg Processes

This subsection provides further detail on the processes comprising the prepreg line. The COSTADE tool [8] was the source of the process models unless noted otherwise.

Load Stringers

Pre-formed, prepreg stringers are manually inserted into cavities in the mold segments.

Assemble Tooling

The mold tool for the fuselage section is built from individual segments. Naturally, one of the production processes is to assemble the segments (with the stringers pre-installed) into a single mandrel tool.

Skin Layup

Automated Fiber Placement (AFP) is used to laminate prepreg tows of unidirectional carbon fiber onto the mandrel tool.

Bag & Prep

After lamination, the fuselage mandrel is prepared for autoclave cure by applying seals, vacuum bags, and performing leak checks.

Autoclave Cure

The bagged mandrel is cured in an autoclave.

De-bag and De-mold

The vacuum bags and seals are removed from, the cured fuselage section is removed from the mandrel and the mandrel is disassembled and returned to the beginning of the production line.

3.2.2 Resin Infusion Processes

This subsection provides further detail on the processes comprising the resin infusion line. The COSTADE tool [8] was the source of the process models unless noted otherwise.

Load Stringers

Dry stringer charges are inserted into cavities in the mandrel tool for the fuselage inner mold line.

Layup Skin

Automated fiber placement is used to laminate tows of bindered dry fiber onto the mandrel tool to form the skin

Locate OML Caul

Caul plates are placed over the laminated skin to form the outer mold line (OML) during the infusion and cure processes.

Bag & Prep

Seals, infusion hardware, and vacuum bags are installed and tested for leaks.

Infuse & Cure

Three processes comprise infuse and cure: compaction steps compress the ply stack, infusion draws the thermoset resin into the dry ply stack, and oven curing which activates the resin. The models for the infusion step of the process were not drawn from COSTADE but rather from FAA publications [9].

3.3 Inputs

For this study, the fuselage dimensional variables and process parameters are fixed as inputs and all of the production line design variables (capacity, inventory, etc.) are left free to be optimized. The required production rate is one of the principle drivers for the cost of the production system and is also left as an input. The inputs in the study are intended to align with data which would be available during early design phases. The input variables in this case are meant to representative only and thus do not describe a particular aircraft design.

Table 1 shows the fuselage geometry variables used in the study. The section of fuselage under study is a straight section with an approximately constant cross-section. The fuselage section resulting from the fabrication has longitudinal stringers co-bonded to a laminated skin. The fuselage is fabricated as a single piece.

Table 1. Early-phase design inputs for a single-piece fuselage.

Description	Dimension (m)
Fuselage Section Length	14.0
Fuselage Diameter	7.00
Average skin thickness	1.98×10^{-2}
Stringer average spacing	0.254
Stringer flattened width	0.406

The process models for both prepreg and the infusion concepts were obtained from publicly available sources. However, the cost data used in the study are only meant to be representative. Capital cost inputs are shown in Table 2.

Table 2. Non-recurring capital costs of capacity for processes of different fabrication methods.

	Cell Capital Costs (notional dollars)					
Prepreg	Load Stringers	Assemble Tooling	Layup Skin	Bag & Prep	Autoclave Cure	Debag & Demold
	25	40	75	10	100	10
Infusion	Load Stringers	Layup Skin	Locate OML Caul	Bag & Prep	Infuse & Cure	Debag & Demold
	25	75	10	10	50	10

The purpose of presenting this study is not to offer a definitive conclusion for which process should be used but rather to demonstrate the framework for analyzing the decision. In principle, a manufacturer interested in analyzing the trade-off in a real situation should replace the general, publicly-available parameters in the models with specific parameters derived from the company's proprietary information.

4. RESULTS

GP constraints are formed from all of the models and optimized to minimize non-recurring costs. A single set of inputs will yield a single cost for each production strategy. Since non-recurring cost is the objective of the optimization, the sensitivities to inputs are the cost drivers of those same inputs. Finally, the costs are compared across a range of required production rate to evaluate the large-scale cost vs rate trade-off for each production method. All the different types of results show that there is no clear-cut answer for the given parameters which production method should be selected. However, the results clearly show the factors which would drive the decision one way or another in a particular case.

4.1 Point Solution Exploration

The speed advantage of the method is used to build a dashboard where point solutions can be explored in near real-time. Figure 5 the dashboard with sliders controlling key inputs, a top-level summary of the cost estimate for the two different production strategies, and detailed information about the optimized production system for each strategy.

As the designer moves the sliders, the inputs to the corresponding fixed variables are updated and the models are optimized again. Top-level results for comparing cost and performance of the two manufacturing techniques are displayed in addition to detailed results regarding the production system. Solving the model takes about 50 milliseconds on a laptop so the designer is able to get near-instant feedback about how changing the design affects the cost of the production systems.

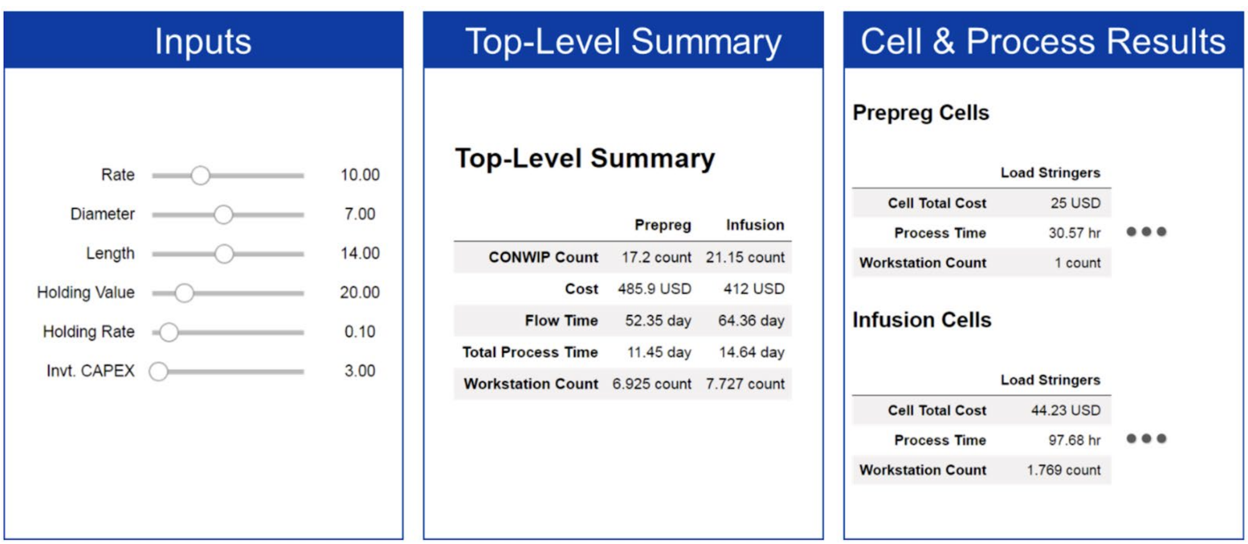

Figure 5. Point-by-point design space explorer for fuselage manufacturing process trade-off.

4.2 Sensitivity / Cost-Driver Analysis

To further compare the two manufacturing techniques, sensitivities for selected variables can be analyzed for the different production techniques. Since the objective function is the total cost, the sensitivity data are useful for identifying the cost drivers for the different manufacturing alternatives. Figure 6 shows a chart of selected sensitivities for the two different production processes.

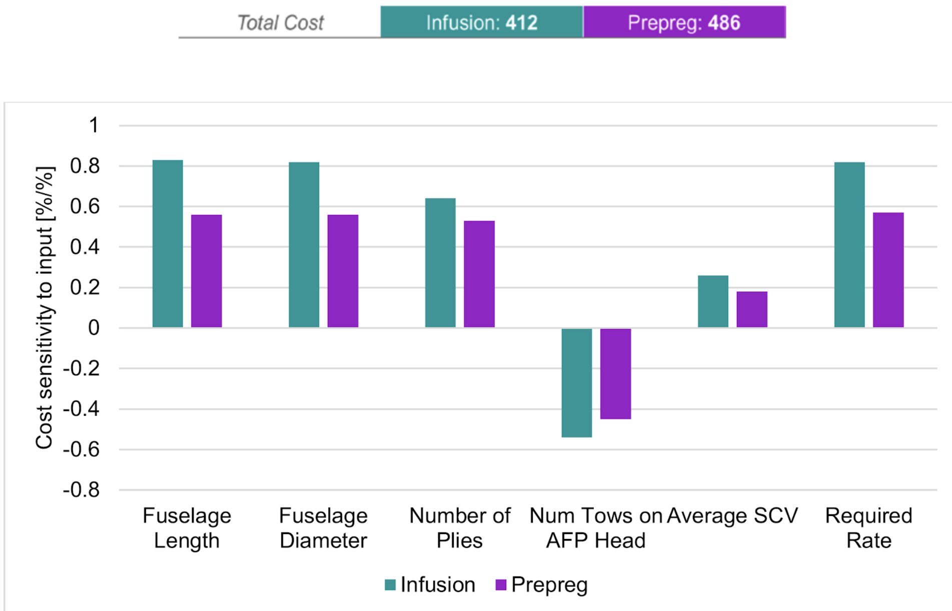

Figure 6. Sensitivity analyses for selected inputs to help compare two different production processes.

Sensitivities are reported in a percent change of the non-recurring cost per percent change of input variable. For example, increasing the diameter of the fuselage by 1% would increase the total cost by 0.82%, this makes sense that increasing the size of the part should increase the cost of the production system. On the other hand, increasing the number of tows on the AFP head would decrease the overall cost of the system as logic would suggest since adding more tows to the AFP machine increases throughput.

At the particular design point (a particular set of inputs) shown in Figure 5, the model predicts a slightly lower cost factory for an infusion process than the corresponding factory for a prepreg process. The designer should not stop and simply choose the apparently cheaper option.

Figure 6 shows, for the same selected fixed variables, the infusion process is more sensitive across the board than the prepreg. This can be used to judge the "volatility" of a concept. Since the sensitivities for the infusion line are higher than the prepreg line, a given perturbation to the inputs will have a more significant impact on the cost of an infusion line. Given that inputs almost always change during the design process, the infusion line appears to be more volatile. If the designers were to select the infusion process only for its lower cost and the fuselage geometry were to change, the cost advantage may quickly be lost.

4.3 Cost and Rate Trade-Off

The previous results are for a single point in the analysis and the sensitivities in the neighborhood of that single point. If the designer wishes to characterize a trade-offs over a wider range of the input, the input can be swept to produce a series of point solutions. Figure 7 shows the costs for a prepreg and an infusion system over a range of required production rates.

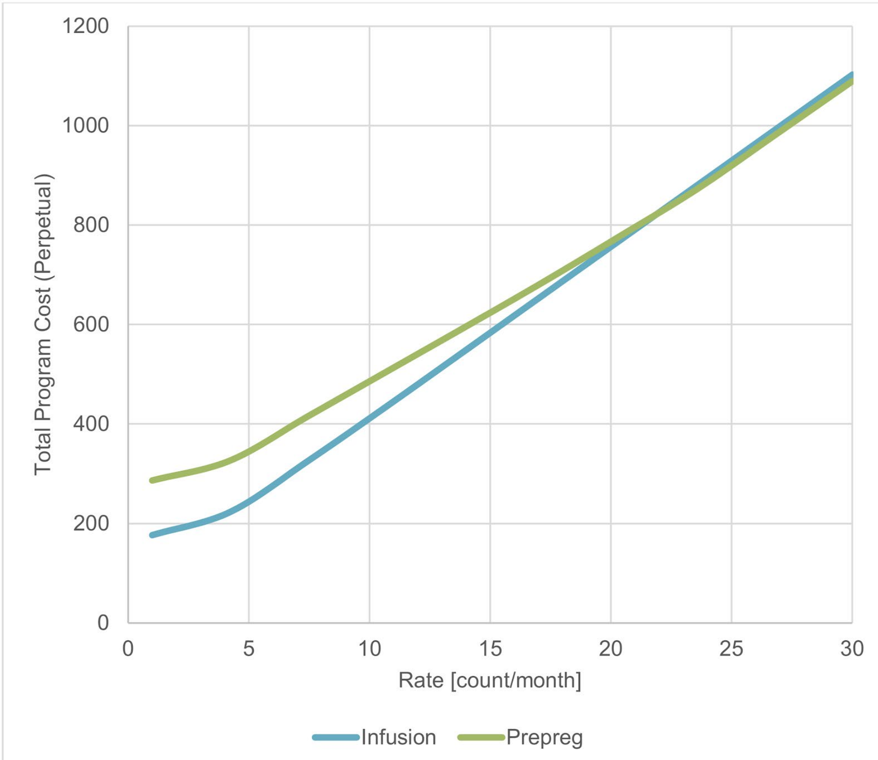

Figure 7. Trade-off analysis showing the effect of required rate on the overall cost of the system for different production methods.

Though the infusion approach starts off less expensive than the prepreg factory, the cost of the infusion factory increases faster than the cost of the prepreg factory as the rate increases until the infusion factory becomes slightly more expensive. This behavior is anticipated by the sensitivity analysis shown in Figure 6 where the sensitivity of the infusion cost to the required rate is higher than that of the prepreg process. Thus, similar behavior is expected for increasing the size of the part or the skin thickness. There is an immense value to the designers to have this behavior so clearly illustrated to know that one choice is not better for all circumstances.

5. DISCUSSION

The case study shows that the method can deliver valuable results with only required inputs which would be available during early design phases. The optimization-based method reveals the couplings between product, manufacturing processes, and production system so the key cost-drivers can be known early on in the design process.

The sensitivity and cost-driver data are valuable in giving a more complete picture of the cost estimate at a particular design point. The sensitivity outputs help to point out where more fidelity should be added to design. If the cost is highly sensitive to a particular and said input is likely to change, obtaining more data through additional design, increased model fidelity, or offline experimentation greatly reduces the likelihood of the cost estimate significantly changing and adds confidence to the decisions based on the model.

In this fashion the method can accompany a maturing design and evolve from the type of large trade-off of this study to more detailed and complex trade-offs. In a sense, the method spans both "top-down" and "bottom-up" modeling applications.

The mathematical framework underlying the method does not require a distinction between "inputs" and "outputs" among variables. Rather, the designer selects which variables in the models are "fixed" or "free" in order to address different questions. In the previous example case study process and fuselage variables were "fixed" but the production system variables were "free" to minimize cost. A different problem could have "free" design variables and "fixed" factory and process variables. This creates the opportunity to use the same core models for many different types of exploration.

The fast solution times and rich trade-off characterizations of the method are extremely effective for communicating and collaborating within multidisciplinary teams. Furthermore, the speed helps to encourage ideas to be tested by lower the barrier to performing "what-if" experiments.

5.1 Future Work and Outlook

The method will be used in the future to model more novel composite fabrication processes, such as thermoplastics, to evaluate the potential impact on cost and performance for aerospace applications. Models for vehicle design optimization can be included to perform co-design of product and production. Finally, application the methods may be expanded to other composites applications outside of aerospace.

6. SUMMARY AND CONCLUSIONS

The rich sensitivity analysis enabled by the presented approach lends support to multi-disciplinary design teams working on complex projects where disciplinary boundaries inhibited communication and innovation.

More generally, this paper shows how manufacturing cost estimation for a product can be modeled to include the unit processes required to create the product and to include the factories where these unit processes are performed in a production line. Moreover, the paper demonstrates that these models can be solved with a non-linear, convex optimization approach that offers fast and reliable solutions.

7. REFERENCES

1. Hueber, C., Horejsi, K. & Schledjewski, R. "Review of cost estimation: methods and models for aerospace composite manufacturing." Advanced Manufacturing: Polymer & Composites Science 2, 1–13 (2016).

2. Curran, R. et al. "Modelling of aircraft manufacturing cost at the concept stage." The International Journal of Advanced Manufacturing Technology 31, 407–420 (2006).

3. Boyd, S., Kim, S.-J., Vandenberghe, L. & Hassibi, A. "A tutorial on geometric programming." Optimization and engineering 8, 67 (2007).

4. Seong, K., Narasimhan, R. & Cioffi, J. M. "Queue proportional scheduling via geometric programming in fading broadcast channels." IEEE Journal on Selected Areas in Communications 24, 1593–1602 (2006).

5. Abbas, W., Perelman, L. S., Amin, S. & Koutsoukos, X. "An Efficient Approach to Fault Identification in Urban Water Networks Using Multi-Level Sensing." Proceedings of the 2Nd ACM International Conference on Embedded Systems for Energy-Efficient Built Environments 147–156 (ACM, 2015). doi:10.1145/2821650.2821666

6. Hoburg, W. & Abbeel, P. "Geometric programming for aircraft design optimization." AIAA Journal (2014).

7. Little, J. D. "A proof for the queuing formula: L= λ W." Operations research 9, 383–387 (1961).

8. Ilcewicz, L. B. et al. "Cost Optimization Software for Transport Aircraft Design Evaluation (COSTADE)—Design Cost Methods [CR-4737]." (1996).

9. Overview of Vacuum-Assisted Resin Transfer Molding Processing. (US Department of Transportation, Federal Aviation Administration, 2013).